Climate Change

Historical context

Fourier (1827)

- Trying to understand what sets the temperature of Earth
- Noticed that the Earth is much colder than would be expected if the sun is the only heat source
- Posits that atmosphere may have something to do with it – relates it to Greenhouse experiments

Tyndall (1861)

- Measured absorption of various gases
- Found that CO_2 and H_2O were strong absorbers of infrared radiation.

Arrhenius (1896)

- Tries to estimate the concentration of various greenhouse gases in the atmosphere
- Uses observations of radiation from the moon
- Derives an ECS to doubling of CO_2.

Energy balance models

Bare rock model

$$E_{in} = E_{out}$$

E_{in} is given by insolation:

$$E_{in} = S_0 \pi r_0^2$$

$$E_{out} = \underbrace{\alpha S_0 \pi r_0^2}_{\text{reflected insolation}} + \underbrace{\sigma T_g^4 \, 4\pi r_0^2}_{\text{blackbody radiation at surface}}$$

$$\Rightarrow \boxed{\frac{S_0(1-\alpha)}{4} = \sigma T_g^4}$$

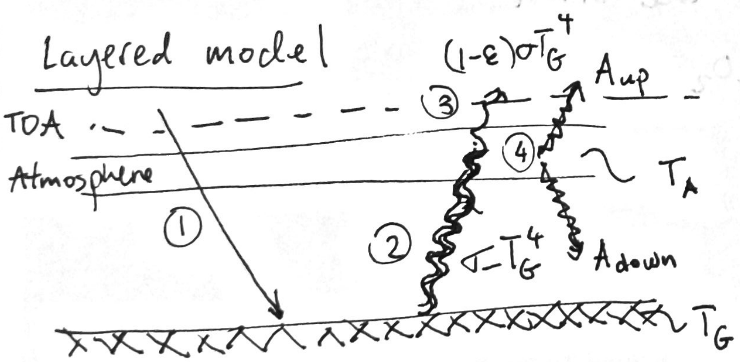
$$\Rightarrow T_g^{\cancel{4}} = \left(\frac{S_0(1-\alpha)}{4}\right)^{1/4} \doteq 255\,K$$

Physical constants

$S_0 \simeq 1360 \ W/m^2$

$\alpha = 0.3$ (Earth)

Layered model



1) Visible radiation from Earth is "invisible" to greenhouse gases and passes through layer

2) Infrared radiation is emitted at Earth's surface ~~at~~ via Stefan-Boltzmann $\sim \sigma T_g^4$

3) Atmosphere GHGs absorb some fraction of ground IR ε, while $(1-\varepsilon)$ passes through

4) Atmosphere emits this radiation upwards and downwards, each at $\varepsilon \sigma T_A^4$.

(Kirchoff's law: emissivity constant is the same for absorption and emission

Layer model continued

- Energy balance at top of atmosphere:

$$\frac{S_0(1-\alpha)}{4} = (1-\varepsilon)\sigma T_G^4 + \varepsilon\sigma T_A^4$$

- Energy balance at surface:

$$\frac{S_0(1-\alpha)}{4} + \varepsilon\sigma T_A^4 = \sigma T_G^4$$

- Can write in terms of emission temperature

$$\Rightarrow \sigma T_G^4 = \frac{S_0(1-\alpha)}{2} - (1-\varepsilon)\sigma T_G^4$$

$$\Rightarrow \sigma T_G^4 = \frac{S_0(1-\alpha)}{2(2-\varepsilon)} = \frac{2}{2-\varepsilon}\,\frac{S_0(1-\alpha)}{4} \qquad \sigma T_E^4 = \frac{S_0(1-\alpha)}{4}$$

$$\Rightarrow \boxed{T_G = \sqrt[4]{\frac{2}{2-\varepsilon}}\;T_E}$$

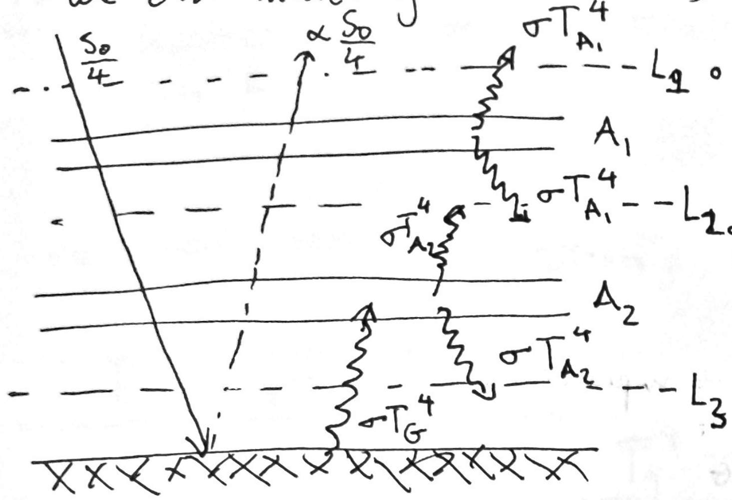
$$\cancel{\frac{S_0(1-\alpha)}{4}} \qquad \varepsilon\sigma T_A^4 = \sigma\frac{2}{2-\varepsilon}T_E^4 - \sigma T_E^4$$

$$\Rightarrow T_A = \sqrt[4]{\frac{1}{2-\varepsilon}}\;T_E = \left(\frac{1}{2}\right)^{1/4} T_G$$

- We can see that the atmosphere is always colder than the ground. Physically, the atmosphere only gets its energy from the ground, but the ground gets radiation from the sun and from the atmosphere emitting.

Multi-layer model

- We can model global warming by adding layers:



- This happens if the atmosphere is so opaque that a single layer will absorb <u>all</u> of the greenhouse gases

- Do an energy budget ~~at~~ between each intermediate layer — it is easy since all of the energy from non-adjacent layers is completely absorbed by the adjacent layers and doesn't factor in.

- ~~For layer L_k~~
- Given n-layers,
  - L_1: $\frac{S_0(1-\alpha)}{4} = \sigma T_{A_1}^4$
  - L_k: $\frac{S_0(1-\alpha)}{4} + \sigma T_{A_{k-1}}^4 = \sigma T_{A_k}^4$
  - L_n: $\frac{S_0(1-\alpha)}{4} + \sigma T_{A_{n-1}}^4 = \sigma T_{AG}^4$

- Since $\frac{S_0}{4}(1-\alpha) = \sigma T_E^4$, this inductively can be solved for σT_G^4:

$$T_G = \sqrt[4]{n}\, T_E .$$

Feedbacks in the layer model

$$T_G = \left(\frac{2}{2-\varepsilon(T)} \frac{S_o(1-\alpha(T))}{4} \right)^{1/4} \qquad (T = T_G)$$

Allow ε and α to be given as functions of temperature

What sort of conditions would result in runaway ~~feedbacks~~? temperatures?

- For positive feedback with emissivity,

$$\frac{d\varepsilon}{dT} > 0$$

write $\varepsilon = f(CO_2, H_2O, \dots)$

Clausius-Clapeyron: $\underbrace{\frac{\partial \varepsilon}{\partial H_2O}}_{>0} \; \underbrace{\frac{\partial H_2O}{\partial T}}_{>0}$

Solubility decreases with T increasing so negative feedback, ~~but~~ this occurs on much slower timescales.

- For albedo, we ~~get~~ would need

$$\frac{d\alpha}{dT} < 0$$

For colder climates, $\frac{d\alpha}{dT} < 0$ (e.g. snowball Earth)

But today, feedback is dominated by clouds: $\frac{d\alpha}{dT} > 0$.

- Multiple Equilibria:
  - Suppose snowball Earth: $\alpha = 0.6$, $\varepsilon = 0.76$ $\rightsquigarrow$ $T_G = 249K$
  - Runaway feedback $\frac{d\alpha}{dT} < 0$
  - New equilibrium ~~T_G~~ $\alpha = 0.3$ $\varepsilon = 0.76$ $T_G = 287\,K$.

Latitudinal Variation

- Insolation is dependent on the latitude
- Let insolation depend on temperature.

$$\alpha(T) = \begin{cases} 0.7 & T < 273 \\ 0.3 & T > 273 \end{cases}$$

- $T = T(\varphi)$.

Budyko Sellers model

$$c_p \frac{\partial T(\varphi)}{\partial t} = S(\varphi)(1-\alpha(\varphi)) - \left(1-\frac{\varepsilon}{2}\right)\sigma T(\varphi)^4 + D\frac{\partial^2 T(\varphi)}{\partial^2 \varphi}$$

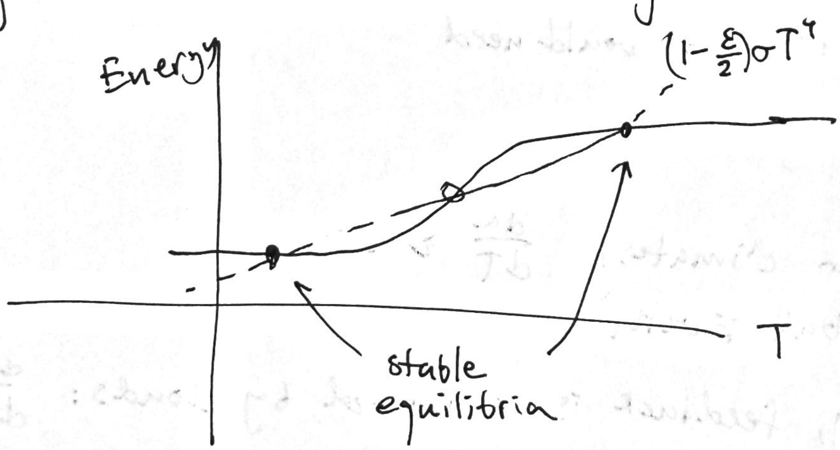
Multiple Equilibria revisited

- Simplest model: uniform temp

$$c_p \frac{dT}{dt} = \cancel{\delta\left(1+\frac{\varepsilon}{2}\right)} S_0 \frac{(1-\alpha)}{4} - \left(1-\frac{\varepsilon}{2}\right)\sigma T^4$$

Assuming α to be some sort of sigmoidal creature

$\rightsquigarrow$



Vertical Structure of the atmosphere: towards convection

Dry adiabatic lapse rate

Assume

1) Ideal gas $p = \rho R T$
2) Hydrostatic balance: $\frac{dp}{dz} = -\rho g$
3) Thermodynamics: $c_v \, dT = -p \, d\alpha + \delta Q$

   Adiabatic: $\delta Q = 0$

Apply ideal gas law 100x to thermodynamic equation:

$$\rightsquigarrow \quad 0 = c_p \, dT - \frac{dp}{\rho} \qquad \rho = \rho_p = \text{parcel density}$$

Assuming HSB in the environment:

$$\frac{dp_E}{dz} = -\rho_E g \qquad \rho_E = \text{environmental density}$$

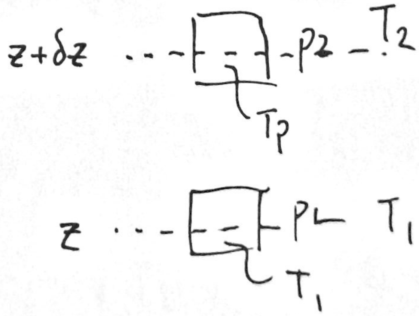
Before any perturbation $\rho_p = \rho_E$

$$\Rightarrow \quad c_p \, dT = \frac{dp}{\rho_p} = \frac{-\rho_E g \, dz}{\rho_p} \approx -g \, dz$$

$$\Rightarrow \quad \boxed{\frac{dT}{dz} = \frac{-g}{c_p} = -\Gamma_d}$$

Dry adiabatic lapse rate for a parcel:
rate that a parcel changes temperature with height under adiabatic displacement.

- Stability
  - Compare environmental density changes to parcel:



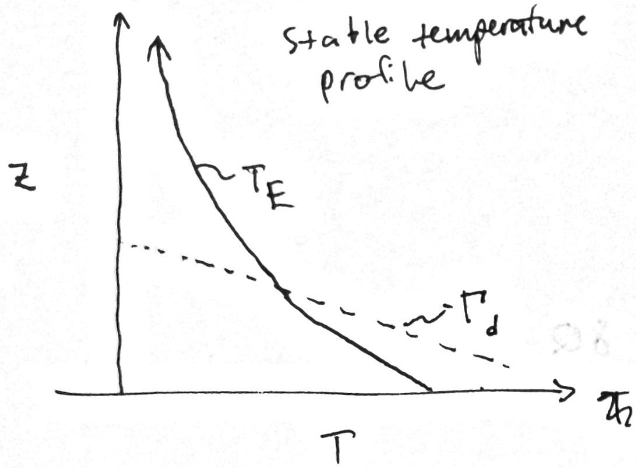
$$T_2 = T_1 + \left(\frac{dT}{dz}\right)_E \delta z \qquad \rho_2 = \frac{p_2}{R T_2}$$

$$T_p = T_1 - \Gamma_d \, dz \qquad \rho_p = \frac{p_2}{R T_p}$$

Stable if $\rho_p > \rho_2 \Leftrightarrow T_p < T_2 \Rightarrow \left(\frac{dT}{dz}\right)_E > -\Gamma_d$

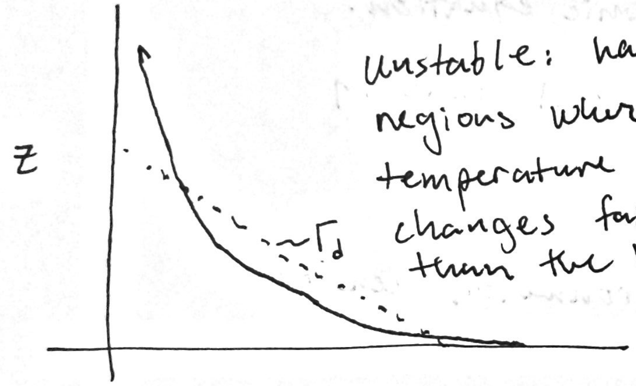
Unstable if $\left(\frac{dT}{dz}\right)_E < -\Gamma_d$.

• Temperature profile

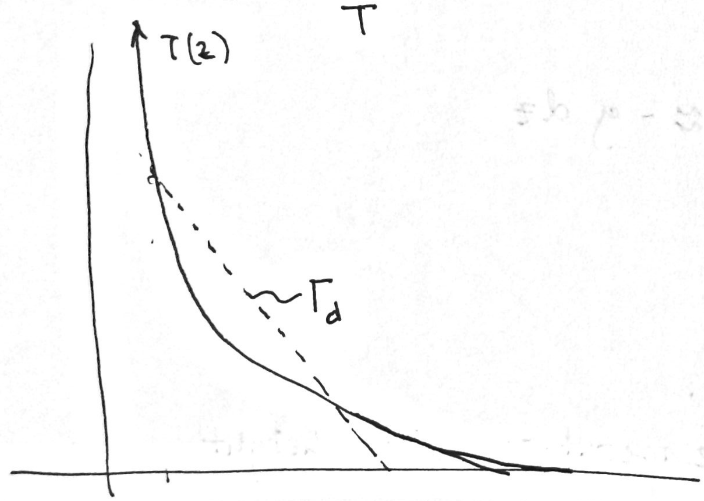


• stable if $\left(\frac{dT}{dz}\right)_E > -\Gamma_d$

• Unstable if $\left(\frac{dT}{dz}\right)_E < -\Gamma_d$



Unstable: has regions where temperature profile changes faster than the lapse rate.



Moist Adiabatic Lapse Rate

- Preliminaries
  - Specific humidity $q = \frac{\rho_v}{\rho_{tot}}$
  - *Saturation specific humidity $q^* = \frac{\rho_v^*}{\rho_{tot}}$
  - vapor pressure e
  - *Saturation vapor pressure e_s

$$* \quad q^* = \frac{\frac{e_s}{R_v T}}{\frac{p}{RT}} = \frac{R}{R_v}\frac{e_s}{p}$$

  - Clausius Clapeyron:

$$e_s = Ae^{\beta T} \qquad [T] \text{ in } {}^\circ C$$

$$\Rightarrow q^* = \frac{R}{R_v}\frac{Ae^{\beta T}}{p}.$$

- Derivation

Need

1) Ideal gas law $p = \rho RT$
2) Hydrostatic balance $\frac{dp}{dz} = -\rho g$
3) Thermodynamics (with latent heat) $0 = c_p\, dT - \frac{1}{\rho} dp + L\, dq$

Also assume $q = q^*$: we are fully saturated

$$\Rightarrow L\,dq = L\,dq^* = L\left[-\frac{R}{R_v}\frac{Ae^{\beta T}}{p^2}dp + \frac{R}{R_v}\frac{Ae^{\beta T}\beta}{p}dT\right]$$

$$= L\left[-\frac{q^*}{p}dp + \beta q^* dT\right]$$

$$\Rightarrow (c_p + L\beta q^*)\,dT = \left(\frac{1}{\rho} + \frac{Lq^*}{p}\right)dp$$

$$= \left(1 + \frac{Lq^*}{RT}\right)\frac{dp}{\rho}$$

$$\Rightarrow \quad \frac{dT}{dz} = \frac{-g}{c_p}\left(\frac{1+\frac{Lq^*}{RT}}{1+\frac{\beta}{c_p}Lq_l^*}\right)$$

$$\frac{dT}{dz} = -\Gamma_d\left(\frac{1+\frac{Lq^*}{RT}}{1+\frac{\beta}{c_p}Lq_l^*}\right)$$

- For Earth $\frac{1}{RT} < \frac{\beta}{c_p}$ so $|\Gamma_s| < |\Gamma_d|$.
- Lapse rate is temperature-dependent and moisture-dependent